EIGHTH PERIOD: Topic 1(g) *2005*

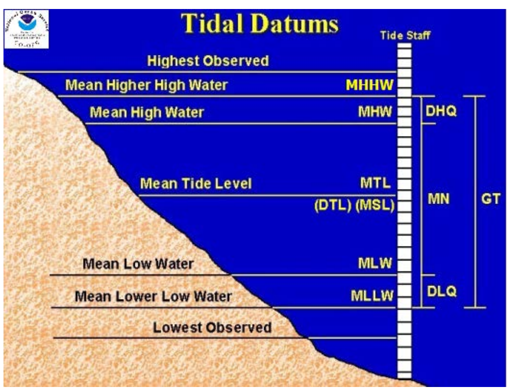
Vertical Reference Frames (Datums) and CCS83 Combined Grid Factors

Reading: Pages 99-108 of "Introduction to Geodesy – The History and Concepts of Modern Geodesy" by James R. Smith.

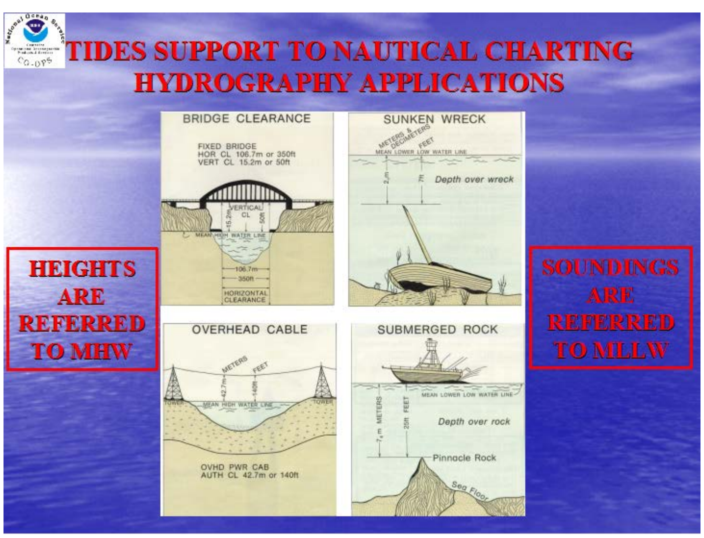
To this point, Topic 1, steps a-f, has dealt with horizontal reference frames and related subjects, including state plane coordinates and California law. Before the last element of the California Coordinate System, the combined grid factor, can be described the topic of vertical datums needs to be discussed.

Vertical Reference Frames (Datums)

Vertical datums are important in that they relate elevations to each other over a particular region. Within regions, there may be many different vertical datums. Different vertical datums have a different "zero surface" that other vertical values are use as their reference. This, in fact, is the definition of a vertical datum: *the zero surface to which elevations or heights are referenced is called a vertical datum.*



The National Geodetic Survey (NGS) defines three categories of vertical datums. They are assumed, tidal, and geodetic. An assumed vertical datum is a datum where the zero surface is arbitrarily picked. Many cities, counties, and other entities have historically defined their own assumed vertical datum. A tidal datum is defined by observation of tidal variations over some period of time. They include mean sea level (MSL), mean lower low water (MLLW), mean low water (MLW), mean high water (MHW), mean higher high water (MHHW), etc. See pisture to the left. A geodetic vertical datum is either directly or loosely based on mean sea level (MSL) at one or more points at some specific date.



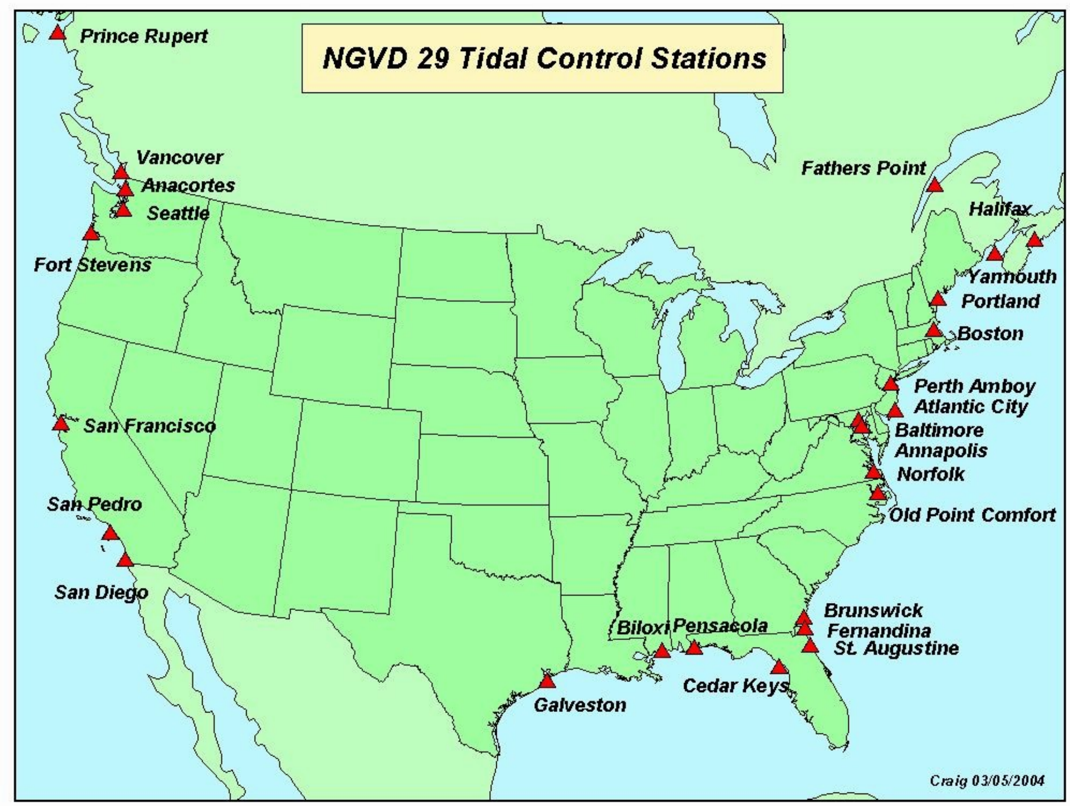
The tidal datums are typically of interest to any entity, such as a city or a local port authority that is adjacent to the coast or building public works projects over tidal areas. The reason is that having a vertical datum that is either, MLLW, MLW, MHW, or MHHW has advantages for making decisions related to shipping and boating concerns. For example, what size ship will fit into a shipping channel or berth, and will a ship clear a public works project such as a bridge to name a few. See picture to the left.

Historically, the most common zero surface used by surveyors is mean seal level (MSL). Interestingly, MSL is a tidal vertical datum that is determined by continuously measuring the rise and fall of the ocean at "tide gauge stations" on seacoasts for a period of about 19 years. This averages out the highs and lows of the tides caused by the changing effects of the gravitational forces from the sun and moon which produce the tides. It turns out that MSL is a close approximation to another surface, defined by gravity, called the **geoid**, which is the *true zero surface for measuring elevations*. The geoid will be discussed later in the lesson.

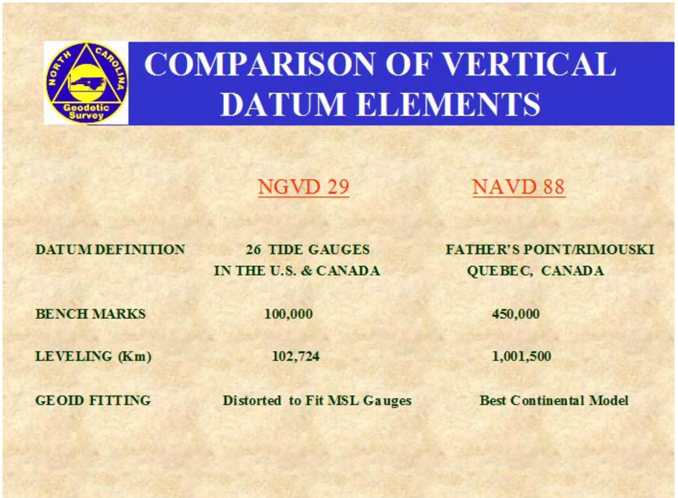
Geodetic Vertical Datums

The two main national geodetic vertical datums, the National Geodetic Vertical Datum of 1929 (NGVD29) and the North American Vertical Datum of 1988 (NAVD88) will be discussed in this section.

The Sea Level Datum of 1929 was a datum where m*ean sea level* was held fixed at the sites of 26 tide gauges, 21 in the U.S.A. and 5 in Canada. The datum is defined by the observed heights of mean sea level at the 26 tide gauges and by the set of elevations of all bench marks resulting from the adjustment. A total of 106,724 km of leveling was involved, constituting 246 closed circuits and 25 circuits at sea level. The datum was not actually mean sea level, the geoid, or any other equipotential surface. Therefore, it was renamed in 1973 to the National Geodetic Vertical Datum of 1929 (NGVD29). The tidal control stations for NGVD29 are shown in the picture below.

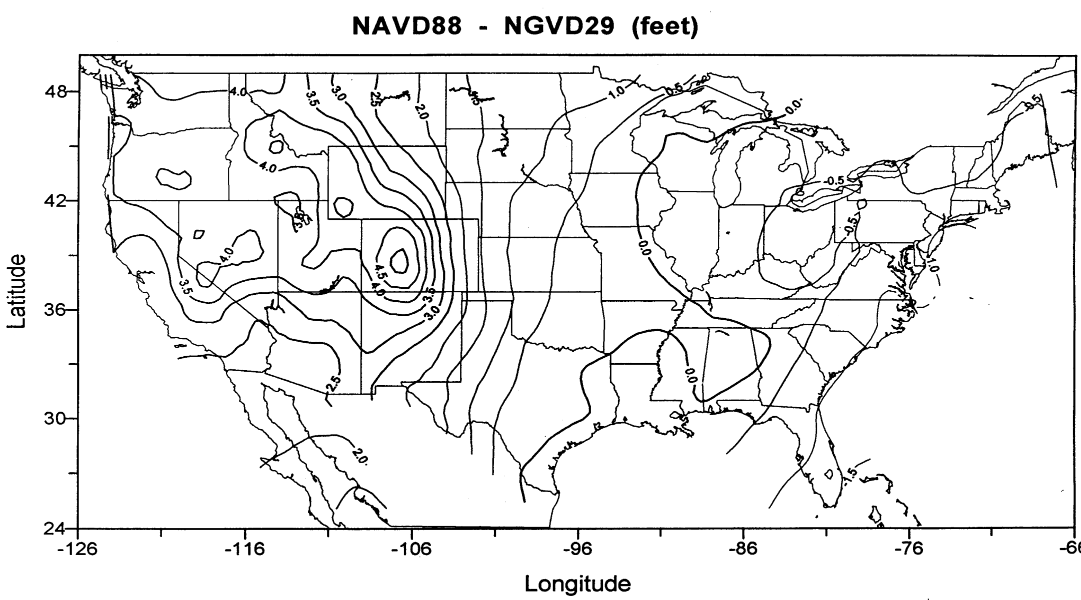


As additional leveling was added to NGVD29 it was discovered that distortions of as much as 9 meters were present when trying to fit the leveling to the held NGVD29 height values. Because of the distortions, a new national vertical datum was created. It is the North American Vertical Datum of 1988.



NAVD88 is the current official vertical datum for the United States. It is the vertical control datum established in 1991 by the minimum-constraint adjustment of the Canadian-Mexican-U.S. leveling observations. It held fixed the height of the primary tidal bench mark, referenced to the new International Great Lakes Datum of 1985 local mean sea level height value, at Father Point/Rimouski, Quebec, Canada (see picture above). Additional tidal bench mark elevations were not used due to the demonstrated variations in sea surface topography, i.e., the fact that mean sea level is not the same equipotential surface at all tidal bench marks.

The picture above shows a comparison of NGVD29 and NAVD88 and the picture below shows the differences in heights between the two vertical datums.



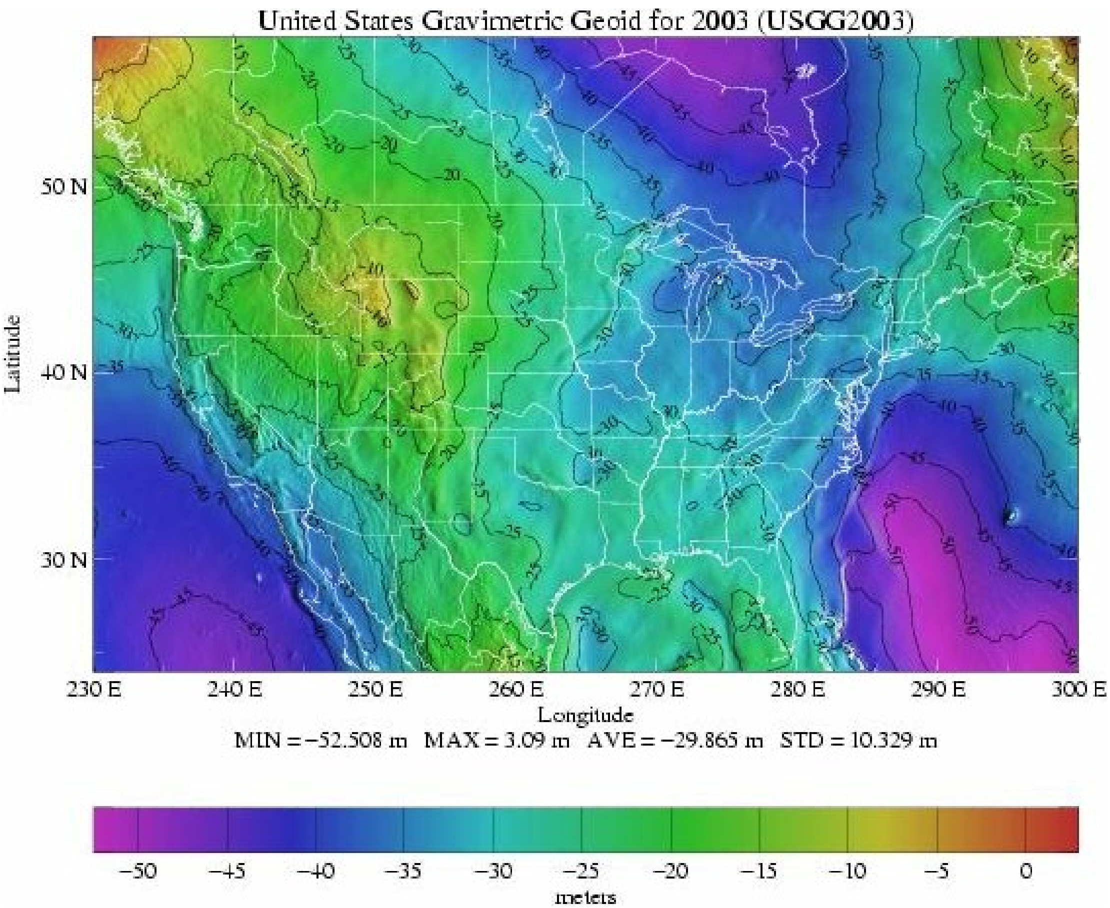
The Geoid and Orthometric Heights, Ellipsoid Heights, and Geoid Heights

The geoid is something that you can’t see or touch and most people have never heard of it. It does, however, allow us to relate heights produced by GPS methods to NAVD88 heights. The geoid is, for all intents and purposes, the same as mean sea level. It is studied by making use of the latest satellite technology, in order to help us measure the Earth we live on. Whether we simply want to know how high we are above sea level when using a handheld GPS receiver or want to build a road faster and cheaper, using the geoid will help.

The geoid surface is described by geoid heights that refer to a suitable earth reference ellipsoid. The following are definitions for the geoid:

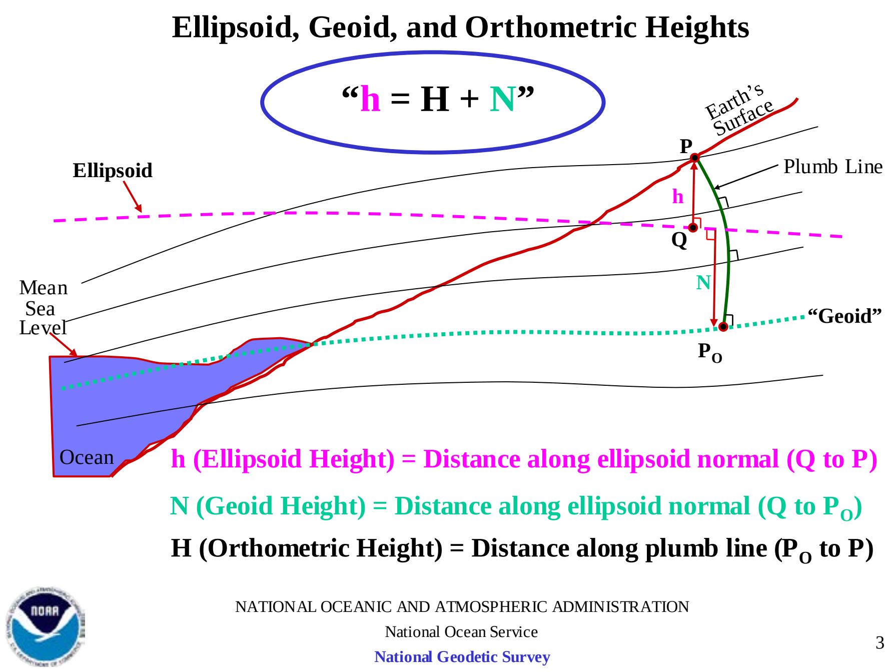
1. *A hypothetical surface of the Earth that would exist if a cross section were taken at sea level. It is perpendicular to the force of gravity at every point.*
2. *The equipotential surface of the Earth's gravity field which best fits, in a least squares sense, global mean sea level.*
3. *The geoid is that equipotential surface of the Earth gravity field that most closely approximates the mean sea surface. At every point the geoid surface is perpendicular to the local plumb line.*

The following picture shows the 2003 geoid contour values for the United States.



The heights measured from the geoid to a point above, on, or below the surface of the earth are called **orthometric heights**. The heights measured from the reference ellipsoid to the geoid are called **geoid heights**. And the heights measured from the reference ellipsoid to the point above, on, or below the surface of the earth are called **ellipsoid heights**.

Geoid heights are derived from a national geoid model produced by NGS. The current geoid model is Geoid03. As NGS obtains more observational information from various sources and methodologies, the geoid model will be improved and a new national geoid model will be released. The relationship of the heights to each other is shown in the picture below.



Notice that the equation h = H + N does not appear to be algebraically correct. However, remember that the ellipsoid heights and geoid heights are measured from the same reference surface, which is the ellipsoid. This means that the geoid heights, as represented in the picture, are actually negative, making the equation correct.

The ellipsoid heights produced by GPS methods are related to the WGS84 reference ellipsoid, which, for our purposes, is the same as the GRS80 reference ellipsoid, which is the basis of NAD83. So, the GPS derived ellipsoids heights along with the geoid heights result in NAVD88 orthometric heights from the above equation.

Now that vertical datums have been described the CCS83 combined grid factors (cgf) can be discussed.

State Plane Coordinates – CCS83 Combined Grid Factors

Distances measured on the earth are different than the corresponding distance on the ellipsoid and the corresponding distance on the grid. The factor for converting a ground distance to a CCS83 zone grid is known as the **combined grid factor (cgf)**. Note: the same concepts apply to CCS27 and other state plane coordinate systems.

The combined grid factor is a combination of two factors. The first is known as the **elevation factor** and is used to reduce the ground distance to an ellipsoid distance. The second is known as the **scale factor** and is used to reduce the ellipsoid distance to a CCS83 zone grid distance. The product of the elevation factor and the scale factor is the combined grid factor. Note: the combined grid factor is also known by the lesser used term, combined scale factor.

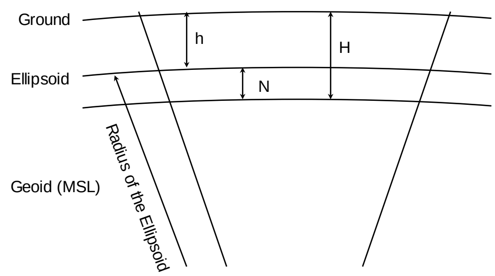
The elevation factor takes into account the radius of curvature of the earth and the ellipsoid height. This factor multiplied by a ground distance reduces the ground distance to an ellipsoid distance. The equation and definitions are in the following diagram. Note that the denominator in the equation is R + N +H. Remember that N + H is the ellipsoid height.

Combined Grid Factor (Combined Scale Factor)

- Elevation Factors
  - Before a Ground Distance can be reduced to the Grid, it must first be reduced to the ellipsoid of reference.

$$EF = \left| \frac{R}{R + N + H} \right|$$

R = Radius of Curvature.
N = Geoidal Separation.
H = Mean Height above Geoid.
h = Ellipsoidal Height



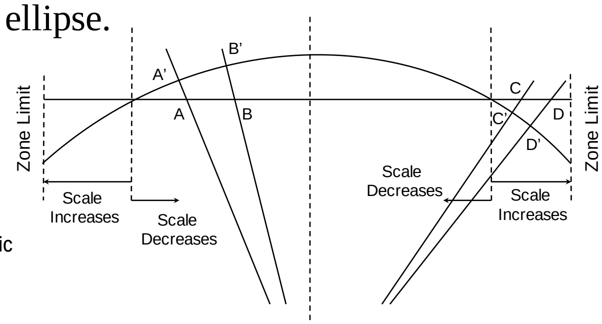
So, by applying the elevation factor to the ground distance a distance on the ellipsoid is obtained. When reducing the ellipsoid distance to a grid distance the scale factor and the ellipsoid distance are multiplied. The following diagram depicts the relationship of ellipsoid distances to grid distances for any of the six CCS83 zones.

Some of the key facts to remember for the scale factor for a CCS83 zone are:

- The scale factor at the standards parallels is 1.
- The scale factor decreases between the standard parallels.
- The scale factor increases external to the standard parallels.
- A distance on the ellipsoid, A’ – B’ on the diagram, between the standard parallels is greater than the corresponding grid distance, A – B on the diagram.
- A distance on the ellipsoid, C’ – D’ on the diagram, external to the standard parallels is smaller than the corresponding grid distance, C – D on the diagram.
- The scale factor is the same along a parallel of latitude.

Combined Grid Factor (Combined Scale Factor)

- A scale factor is the Ratio of a distance on the grid projection to the corresponding distance on the ellipse.



- Grid Distance A-B is smaller than Geodetic Distance A'-B'.
- Grid Distance C-D is larger than Geodetic Distance C'-D'.

Ok, now we will address the computational method of converting a ground distance to a grid distance and the reverse, show an example, and then give a problem to solve. The computational method, like other computations for the CCS83 system, makes use of the polynomial coefficient tables. They are listed at the end of this lesson.

Step one: The first step in calculating the elevation factor is to calculate the radius of curvature of the ellipsoid. This is computed from the equation below, which utilizes two tabled contants.

Converting Measured Ground Distances to Grid Distances

- Determine Radius of Curvature of the Ellipsoid: R_a

$$R_a = r_0/k_0$$

R_a = geometric mean radius of curvature of the ellipsoid at the projection origin

r_0 = geometric mean radius of the ellipsoid at the projection origin, scaled to grid (tabled constant)

k_0 = grid scale factor of the central parallel (tabled constant)

Step Two: Step two uses the result from step one to directly calculate the elevation factor.

Converting Measured Ground Distances to Grid Distances

- Determine the Elevation Factor: r_e

$$r_e = R_a / (R_a + N + H)$$

r_e = elevation factor

R_a = radius of curvature of the ellipsoid

N = geoid separation

H = elevation

Step 3: Step 3 utilizes the radial distance (u) discussed in previous steps and tabled constants to compute the scale factor.

Converting Measured Ground Distances to Grid Distances

- Determine the Point Scale Factor: k

$$k = F_1 + F_2u^2 + F_3u^3$$

k = point scale factor

u = radial difference

F_1, F_2, F_3 = polynomial coefficients (tabled constants)

Step 4: In the fourth step, the cgf is computed.

Converting Measured Ground Distances to Grid Distances

- Determine the Combined Grid Factor: cgf

$$cgf = r_e\, k$$

cgf = combined grid factor

r_e = elevation factor

k = point scale factor

Step 5 (a): In step 5 the cgf is applied to a ground distance to get the corresponding grid distance.

Converting Measured Ground Distances to Grid Distances

- Determine Grid Distance

$$G_{grid} = cgf(G_{ground})$$

Note: G_{ground} is a horizontal ground distance

Step 5 (b): Or the grid distance is divided by the cgf to get the corresponding horizontal ground distance.

Converting Grid Distances to Horizontal Ground Distances

- Determine Ground Distance

$$G_{ground} = G_{grid}/cgf$$

Example:

Example

In CCS83 Zone 1 from station "Me" to station "You" you have a measured horizontal ground distance of 909.909m. Stations Me and You have elevations of 3333.333m and a geoid separation 0f -30.5m. Compute the horizontal grid distance from Me to You. (To calculate the point scale factor assume u = 15555.000)

Step 1:

Example

- Determine Radius of Curvature of the Ellipsoid: R_a

$R_a = r_0/k_0$

$R_a = 6374328/0.999894636561$

$R_a = 6374999.69189$

Step 2:

Example

- Determine the Elevation Factor: r_e

$r_e = R_a/(R_a + N + H)$

$r_e = 6374999.69189/(6374999.69189 – 30.5 + 3333.333)$

$r_e = 0.9994821768$

Step 3:

Example

- Determine the Point Scale Factor: k

$k = F_1 + F_2u^2 + F_3u^3$

$k = 0.999894636561 + 1.23062E\text{-}14(15555)^2$

$+ 5.47E\text{-}22(15555)^3$

$k = 0.9998976162$

Step 4:

Example

- Determine the Combined Grid Factor: cgf

$cgf = r_e\, k$

$cgf = 0.9994821768(0.9998976162)$

$cgf = 0.999379846$

Step 5:

Example

- Determine Grid Distance

$G_{grid} = cgf(G_{ground})$

$G_{grid} = 0.999379846(909.909)$

$G_{grid} = 909.3447$

Problem:

Problem

In CCS83 Zone 4 from station “here” to station “there” you have a measured horizontal ground distance of 1234.567m. Station here and there have elevations of 2222.222m and a geoid separation 0f -30.5m. Compute the horizontal grid distance from here to there. (To calculate the point scale factor assume u = 35000)

The answer to the problem is at the end of the lesson.

Summary: This concludes the geodesy topic for period 8. Many of the concepts are fundamental to understanding and correctly utilizing GPS. The next topic in this period will address GPS.

California Coordinate System 1983 (CCS83)

CALIFORNIA ZONE 1, CA01, ZONE# 0401

The customary limits of the zone are from 39° 20' N to 42° 20' N.

Meters

B_s = 40° 00' N
B_n = 41° 40' N
B_b = 39° 20' N
L_o = 122° 00' W
N_b = 500000.0000 m
E_o = 2000000.0000 m
B_o = 40.8351061249° N
$SinB_o$ = 0.653884305400
R_b = 7556554.6408 m
R_o = 7389802.0597 m
N_o = 666752.5811 m
K = 12287826.3052 m
k_o = 0.999894636561
M_o = 6362067.2798 m
r_o = 6374328. m
L_1 = 111039.0203
L_2 = 9.65524
L_3 = 5.63491
L_4 = 0.021275
G_1 = 9.005843038E-06
G_2 = -7.05240E-15
G_3 = -3.70393E-20
G_4 = -1.1142E-27
F_1 = 0.999894636561
F_2 = 1.23062E-14
F_3 = 5.47E-22

US Survey Feet

B_s = 40° 00' N
B_n = 41° 40' N
B_b = 39° 20' N
L_o = 122° 00' W
N_b = 1640416.667 ft
E_o = 6561666.667 ft
B_o = 40.8351061249° N
$SinB_o$ = 0.653884305400
R_b = 24791796.351 ft
R_o = 24244708.924 ft
N_o = 2187504.093 ft
K = 40314310.136 ft
k_o = 0.999894636561
M_o = 20872882.401 ft
r_o = 20913107.780 ft
L_1 = 364300.5191
L_2 = 31.6772
L_3 = 18.4872
L_4 = 0.069800
G_1 = 2.744986448E-06
G_2 = -6.55192E-16
G_3 = -1.04884E-21
G_4 = -9.6167E-30
F_1 = 0.999894636561
F_2 = 1.14329E-15
F_3 = 1.55E-23

California Coordinate System 1983 (CCS83)
CALIFORNIA ZONE 2, CA02, ZONE# 0402

The customary limits of the zone are from 37° 40' N to 40° 30' N.

Meters

B_s = 38° 20' N
B_n = 39° 50' N
B_b = 37° 40' N
L_o = 122° 00' W
N_b = 500000.0000 m
E_o = 2000000.0000 m
B_o = 39.0846839219° N
$SinB_o$ = 0.630468335285
R_b = 8019788.9307 m
R_o = 7862381.4027 m
N_o = 657407.5280 m
K = 12520351.6538 m
k_o = 0.999914672977
M_o = 6360268.3937 m
r_o = 6373169. m
L_1 = 111007.6240
L_2 = 9.54628
L_3 = 5.63874
L_4 = 0.019988
G_1 = 9.008390180E-06
G_2 = -6.97872E-15
G_3 = -3.71084E-20
G_4 = -1.0411E-27
F_1 = 0.999914672977
F_2 = 1.23106E-14
F_3 = 5.14E-22

US Survey Feet

B_s = 38° 20' N
B_n = 39° 50' N
B_b = 37° 40' N
L_o = 122° 00' W
N_b = 1640416.667 ft
E_o = 6561666.667 ft
B_o = 39.0846839219° N
$SinB_o$ = 0.630468335285
R_b = 26311590.850 ft
R_o = 25795162.985 ft
N_o = 2156844.531 ft
K = 41077187.051 ft
k_o = 0.999914672977
M_o = 20866980.555 ft
r_o = 20909305.294 ft
L_1 = 364197.5131
L_2 = 31.3198
L_3 = 18.4998
L_4 = 0.065577
G_1 = 2.745762818E-06
G_2 = -6.48347E-16
G_3 = -1.05080E-21
G_4 = -8.9858E-30
F_1 = 0.999914672977
F_2 = 1.14370E-15
F_3 = 1.46E-23

California Coordinate System 1983 (CCS83)
CALIFORNIA ZONE 3, CA03, ZONE# 0403

The customary limits of the zone are from 36° 30' N to 39° 00' N.

Meters	US Survey Feet
B_s = 37° 04' N	B_s = 37° 04' N
B_n = 38° 26' N	B_n = 38° 26' N
B_b = 36° 30' N	B_b = 36° 30' N
L_o = 120° 30' W	L_o = 120° 30' W
N_b = 500000.0000 m	N_b = 1640416.667 ft
E_o = 2000000.0000 m	E_o = 6561666.667 ft
B_o = 37.7510694363° N	B_o = 37.7510694363° N
$SinB_o$ = 0.612232038295	$SinB_o$ = 0.612232038295
R_b = 8385775.1723 m	R_b = 27512330.711 ft
R_o = 8246930.3684 m	R_o = 27056804.050 ft
N_o = 638844.8039 m	N_o = 2095943.327 ft
K = 12724574.9735 m	K = 41747209.726 ft
k_o = 0.999929178853	k_o = 0.999929178853
M_o = 6358909.6841 m	M_o = 20862522.855 ft
r_o = 6372292. m	r_o = 20906428.003 ft
L_1 = 110983.9104	L_1 = 364119.7127
L_2 = 9.43943	L_2 = 30.9692
L_3 = 5.64142	L_3 = 18.5086
L_4 = 0.019048	L_4 = 0.062493
G_1 = 9.010315015E-06	G_1 = 2.746349509E-06
G_2 = -6.90503E-15	G_2 = -6.41501E-16
G_3 = -3.71614E-20	G_3 = -1.05230E-21
G_4 = -9.8819E-28	G_4 = -8.5291E-30
F_1 = 0.999929178853	F_1 = 0.999929178853
F_2 = 1.23137E-14	F_2 = 1.14398E-15
F_3 = 4.89E-22	F_3 = 1.38E-23

California Coordinate System 1983 (CCS83)
CALIFORNIA ZONE 4, CA04, ZONE# 0404

The customary limits of the zone are from 35° 20' N to 38° 00' N.

Meters	**US Survey Feet**
B_s = 36° 00' N	B_s = 36° 00' N
B_n = 37° 15' N	B_n = 37° 15' N
B_b = 35° 20' N	B_b = 35° 20' N
L_o = 119° 00' W	L_o = 119° 00' W
N_b = 500000.0000 m	N_b = 1640416.667 ft
E_o = 2000000.0000 m	E_o = 6561666.667 ft
B_o = 36.6258593071° N	B_o = 36.6258593071° N
$\sin B_o$ = 0.596587149880	$\sin B_o$ = 0.596587149880
R_b = 8733227.3793 m	R_b = 28652263.494 ft
R_o = 8589806.8935 m	R_o = 28181724.783 ft
N_o = 643420.4858 m	N_o = 2110955.377 ft
K = 12916986.0281 m	K = 42378478.327 ft
k_o = 0.999940761703	k_o = 0.999940761703
M_o = 6357772.8978 m	M_o = 20858793.249 ft
r_o = 6371557. m	r_o = 20904016.591 ft
L_1 = 110964.0696	L_1 = 364054.6183
L_2 = 9.33334	L_2 = 30.6211
L_3 = 5.64410	L_3 = 18.5174
L_4 = 0.018382	L_4 = 0.060308
G_1 = 9.011926076E-06	G_1 = 2.746840562E-06
G_2 = -6.83121E-15	G_2 = -6.34643E-16
G_3 = -3.72043E-20	G_3 = -1.05351E-21
G_4 = -9.4223E-28	G_4 = -8.1324E-30
F_1 = 0.999940761703	F_1 = 0.999940761703
F_2 = 1.23168E-14	F_2 = 1.14427E-15
F_3 = 4.70E-22	F_3 = 1.33E-23

California Coordinate System 1983 (CCS83)
CALIFORNIA ZONE 5, CA05, ZONE# 0405

The customary limits of the zone are from 33° 30' N to 36° 20' N.

Meters

B_s = 34° 02' N
B_n = 35° 28' N
B_b = 33° 30' N
L_o = 118° 00' W
N_b = 500000.0000 m
E_o = 2000000.0000 m
B_o = 34.7510553142° N
$\sin B_o$ = 0.570011896174
R_b = 9341756.1389 m
R_o = 9202983.1099 m
N_o = 638773.0290 m
K = 13282624.8345 m
k_o = 0.999922127209
M_o = 6355670.9697 m
r_o = 6370113. m
L_1 = 110927.3840
L_2 = 9.12439
L_3 = 5.64805
L_4 = 0.017445
G_1 = 9.014906468E-06
G_2 = -6.68534E-15
G_3 = -3.72796E-20
G_4 = -8.6394E-28
F_1 = 0.999922127209
F_2 = 1.23221E-14
F_3 = 4.41E-22

US Survey Feet

B_s = 34° 02' N
B_n = 35° 28' N
B_b = 33° 30' N
L_o = 118° 00' W
N_b = 1640416.667 ft
E_o = 6561666.667 ft
B_o = 34.7510553142° N
$\sin B_o$ = 0.570011896174
R_b = 30648744.932 ft
R_o = 30193453.753 ft
N_o = 2095707.846 ft
K = 43578078.311 ft
k_o = 0.999922127209
M_o = 20851897.173 ft
r_o = 20899279.068 ft
L_1 = 363934.2590
L_2 = 29.9356
L_3 = 18.5303
L_4 = 0.057234
G_1 = 2.747748987E-06
G_2 = -6.21091E-16
G_3 = -1.05565E-21
G_4 = -7.4567E-30
F_1 = 0.999922127209
F_2 = 1.14477E-15
F_3 = 1.25E-23

California Coordinate System 1983 (CCS83)
CALIFORNIA ZONE 6, CA06, ZONE# 0406

The customary limits of the zone are from 32° 10' N to 34° 30' N.

Meters

B_s = 32° 47' N
B_n = 33° 53' N
B_b = 32° 10' N
L_o = 116° 15' W
N_b = 500000.0000 m
E_o = 2000000.0000 m
B_o = 33.3339229447° N
SinB_o = 0.549517575763
R_b = 9836091.7896 m
R_o = 9706640.0762 m
N_o = 629451.7134 m
K = 13602026.7133 m
k_o = 0.999954142490
M_o = 6354407.2007 m
r_o = 6369336. m
L_1 = 110905.3274
L_2 = 8.94188
L_3 = 5.65087
L_4 = 0.016171
G_1 = 9.016699372E-06
G_2 = -6.55499E-15
G_3 = -3.73318E-20
G_4 = -8.2753E-28
F_1 = 0.999954142490
F_2 = 1.23251E-14
F_3 = 4.15E-22

US Survey Feet

B_s = 32° 47' N
B_n = 33° 53' N
B_b = 32° 10' N
L_o = 116° 15' W
N_b = 1640416.667 ft
E_o = 6561666.667 ft
B_o = 33.3339229447° N
SinB_o = 0.549517575763
R_b = 32270577.813 ft
R_o = 31845868.317 ft
N_o = 2065126.163 ft
K = 44625982.642 ft
k_o = 0.999954142490
M_o = 20847750.958 ft
r_o = 20896729.860 ft
L_1 = 363861.8950
L_2 = 29.3368
L_3 = 18.5396
L_4 = 0.053054
G_1 = 2.748295465E-06
G_2 = -6.08981E-16
G_3 = -1.05713E-21
G_4 = -7.1424E-30
F_1 = 0.999954142490
F_2 = 1.14504E-15
F_3 = 1.18E-23

Solution to Problem

R_a = 6371934.463

r_e = 0.999656153

k = 0.999955870

cgf = 0.999612038

G_{grid} = 1234.088m